Tentamen: Subatomaire fysica

Donderdag 1 maart 2001

naam: ______________________________ **studentno.** ____________________

1. (6 pt.) Parity conservation/violation.
   a) The pseudoscalar $\eta(547)$ meson is observed to decay to 3-pion final states. Explain, why the decay $\eta \to \pi^+\pi^-$ and $\eta \to \pi^0\pi^0$ have never been observed.

   b) Show that the existence of the decays $K^+ \to \pi^+\pi^0$ and $K^+ \to \pi^+\pi^+\pi^-$ implies that parity is violated.

2. (6 pt.) Argue why the nuclear mass surface M(A,Z) has a parabolic form. Sketch the curves M(A=const,Z) for odd A and even A and indicate the decay processes to reach the stable isotopes.

3. (10 pt.) Consider a nuclear system built from 2 nucleons.

a) Write down the possible isospin wavefunctions for the coupling of 2 nucleons to the di-nucleon system. (use the notation $|I, I_3>$ for the isospin I and 3-component I_3)

b) What is the possible spin (J) assignment for the isospin-triplet and -singlet states, respectively?

c) Determine the magnetic dipole moment μ/μ_N of the deuteron for a pure $^3S_1$ state. ($g_l = 1$; g_s = 5.586 for the proton; $g_l = 0$; g_s = -3.826 for the neutron)
The calculated value is about 2.5% larger than the observed value. What could be the reason?

d) Which symmetry principle determines the allowed admixture to the deuteron ground state? Motivate the most likely admixture!

4. (6 pt.) Spin-orbit coupling.
a) Express $\vec{l} \cdot \vec{s}$ in terms of j, l and s. and show that the energy separation of a nuclear spin-orbit doublet is proportional to $2l+1$.

b) In the shell model the radial dependence of the nuclear density is assumed to be of the Woods-Saxon shape.
Obtain an expression for the spin-orbit potential and sketch the radial dependence for $j=l\pm\frac{1}{2}$.

5. (14 pt.) Single-particle shell model (see appendix A).
a) The low-lying levels of $^{39}_{20}Ca$ have spin-parity values, starting from the ground state, of $\frac{3}{2}^+$, $\frac{1}{2}^+$, $\frac{7}{2}^-$, and $\frac{3}{2}^-$. Interpret these values on the basis of the SPSM.

b)The magnetic moment for a nucleus with spin J is given by
$\mu_J = J\ g_J\ \mu_N$ with
$g_J\ =\ g_l\ \pm \frac{g_s - g_l}{2l+1}$ and
$g_l = 1$; g_s = 5.586 for the proton;
$g_l = 0$; g_s = -3.826 for the neutron.
Calculate the ground-state magnetic moment (in units of μ_N) of $^{39}_{19}$K which has a spin value of $\frac{3}{2}$.

c) Argue, why the ground and first excited state of $^{207}_{82}$Pb have spin^{parity} $\frac{1}{2}^-$ and $\frac{5}{2}^-$, respectively.

d) Calculate the ground-state magnetic moment of $^{207}_{82}$Pb (in units of μ_N).

e) The first excited state lies 570 keV above the ground state and decays by γ emission with a halflife of 130 10^{-12} s. What are the possible multipolarities and what is the most likely character of the electromagnetic radiation?

f) The decay width Γ ($\Gamma\tau = \hbar$; $\hbar c$=197.32 MeV fm; $c \approx 3\ 10^8$m/s = $3\ 10^{23}$fm/s) for electromagnetic radiation is given in the single-particle model for E_γ in MeV by:
$\Gamma(E1) = 0.070\ E_\gamma^3\ A^{2/3}$ eV
$\Gamma(M1) = 0.021\ E_\gamma^3$ eV
$\Gamma(E2) = 4.9\ 10^{-8}\ E_\gamma^5\ A^{4/3}$ eV.
Calculate the halflife of the first excited state in $^{207}_{82}$Pb and argue whether this is a good single-particle state.

6. (4 pt.) γ transitions.
For the following γ transitions, name the permitted multipoles and indicate which multipole might be the most intense in the emitted radiation:
a) $\frac{9}{2}^+ \rightarrow \frac{7}{2}^+$

b) $\frac{1}{2}^- \rightarrow \frac{7}{2}^-$

7. (8 pt.) β decay.
a) Supply the missing component(s) in the following processes (the element sequence is H, He, Li, Be, B, C, N):
(1) $\bar{\nu} + {}^3$He $\rightarrow$
(2) ^{6}He $\rightarrow {}^6$Li + β^- +
(3) e^- + ^{8}B $\rightarrow$
(4) ν + ^{12}C $\rightarrow$

b) Sketch and motivate the general behaviour of the momentum spectrum of β particles for 3 cases:
(1) neglecting the correction due to the Coulomb field of the final nucleus;
(2) including the Coulomb correction for β^-;
(3) including the Coulomb correction for β^+.

8. (8 pt.) β decay and lepton helicity.
a) For a $0^+ \rightarrow 0^+$ β^+ decay consider the helicity of the emitted β^+ and ν, sketch directions of spin and momentum and deduce whether the β^+ and the ν tend to be emitted parallel or antiparallel to one another.

b) Repeat this consideration for a $1^+ \rightarrow 0^+$ β^+ decay.

c) What can you say about the recoil energy of the final nucleus for cases a) and b)?

9. (6 pt.) Baryon masses.
In the quark model the baryon masses can be predicted from the sum of the constituent quark masses and a hyperfine interaction dependent on spin ($\vec{s}$) and mass (m) of the constituent quarks:
$m(q_1, q_2, q_3) = m(q_1) + m(q_2) + m(q_3) + \sum_{i<j} a \frac{\vec{s}_i \cdot \vec{s}_j}{m(q_i) m(q_j)}$.
a) Calculate the interaction parameter a from the known nucleon mass $m_N = 939$ MeV/c^2 and the constituent masses $m_u = m_d = 363$ MeV/c^2 and $m_s = 538$ MeV/c^2.
b) Calculate the mass of the Ω^- baryon (see appendix B)

10. (8 pt.) Mass relation in U-spin multiplet.
The mass M of a particular U-spin state $|U, U_3>$ ($M = <U, U_3|H|U, U_3>$) may be considered as a constant term m_0 equal for all members of a multiplet, a term m_s equal for all U-spin members with the same U, and a term m_v proportional to U_3; then $M = m_0 + m_s + m_v$; predict the mass relation for the neutral members of the $J^P = \frac{1}{2}^+$ baryon octet (see appendix B). The U-spin singlet state is given by $\frac{1}{2}(\sqrt{3}\Sigma^0 - \Lambda^0)$.(first, derive the $U_3 = 0$ -state of the U-spin triplet)

11. (4 pt.) Baryon resonances.
In comparison to π^+p scattering, the cross section for π^-p scattering shows additional structure. How can you explain this and what is the relative weight of the contributing amplitudes? (use appendix C)

Total number of points: 80
Your score: ________

Your grade: ________